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Quantum Retrodiction in Non-Equilibrium Thermo Field Dynamics

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The quantum retrodiction for open systems which obey the quantum Markovian dynamics is investigated by means of non-equilibrium thermo Field dynamics (NETFD) which can easily derive the retrodictive time-evolution generators. NETFD can formulate the quantum retrodiction for open systems in the same way as that for closed systems.

KEY WORDS: retrodiction; prediction; non-equilibrium thermo field dynamics.

1. INTRODUCTION

In quantum mechanics, a system which is prepared in a quantum state at initial time t_p evolves with time, and then quantum measurement is performed on the system at later time t_m ($t_m > t_p$). Quantum mechanics predicts the measurement outcome with the probability (Helstrom, 1976), which is referred to as the quantum prediction. Once the measurement outcome is obtained, using the Bayes theorem, one can retrodict the quantum state in which the system was prepared at the initial time t_p. This is referred to as the quantum retrodiction. Although the quantum retrodiction is quite different from the quantum prediction, it can be formulated in the similar way to the quantum prediction (Barnett *et al.*, 2000a,b,c, 2001; Jedrkiewicz *et al.*, 2004; Pegg *et al.*, 2002a,b; Pegg and Jeffers, 2005; Pegg, 2006). In the quantum retrodiction, the system which is prepared in a quantum state at the measurement time t_m evolves backward with time and quantum measurement is performed on the system at the time t_p. The probability that the quantum measurement carried out at the time t_p yields the outcome is equal to that obtained by the quantum prediction in combined with the Bayes theorem (Barnett *et al.*, 2000b). The quantum retrodiction is useful for investigating quantum communication systems (Barnett *et al.*, 2000a; Jedrkiewicz *et al.*, 2004), where a receiver must infer which quantum state a sender prepared. For a closed system,

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the time-evolution of which is unitary, the retrodictive time-evolution can easily be obtained. The retrodictive time evolution of open systems was obtained by Pegg and Barnett when the predictive time evolution obeys the quantum Markovian dynamics (Barnett *et al.*, 2001; Pegg *et al.*, 2002b). This paper investigates the retrodictive quantum dynamics for open systems within the framework of non-equilibrium thermo field dynamics (NETFD) (Arimitsu and Umezawa, 1985, 1987a,b; Umezawa *et al.*, 1982; Umezawa, 1993). Using NETFD, one can formulate the quantum retrodiction of open systems in the same way as that for closed systems and thus one can easily obtain the retrodictive time-evolution generator.

2. NON-EQUILIBRIUM THERMO FIELD DYNAMICS

A quantum state is described by a density matrix ρ defined on a Hilbert space H, which satisfies $\rho > 0$ and $\mathrm{Tr}\rho = 1$. In NETFD, a quantum state is given by a state vector which belongs to an tensor product Hilbert $\mathscr{H} \otimes \tilde{\mathscr{H}}$, where $\tilde{\mathscr{H}}$ is an auxiliary Hilbert space with the same dimension as that of $\mathscr{H}$. Any operator A defined on $\mathscr{H}$ is related with the corresponding operator $\tilde{A}$ defined on $\tilde{\mathscr{H}}$ by the tilde conjugation which satisfies (Umezawa *et al.*, 1982; Umezawa, 1993)

$$(A_1 A_2)\tilde{} = \tilde{A}_1 \tilde{A}_2, \tag{1}$$

$$(c_1 A_1 + c_2 A_2)\tilde{} = c_1^* \tilde{A}_1 + c_2^* \tilde{A}_2, \tag{2}$$

$$(A^\dagger)\tilde{} = \tilde{A}^\dagger, \tag{3}$$

$$(\tilde{A})\tilde{} = \sigma_A A, \tag{4}$$

where c_k is a c-number and σ_A is a phase factor with $\sigma_A = 1$ for a bosonic operator A. For any operator A defined on $\mathscr{H}$, there exists a unique vector $|A\rangle\rangle$ belonging to $\mathscr{H} \otimes \tilde{\mathscr{H}}$, and for any vector $|A\rangle\rangle$, there is a unique operator A defined on $\mathscr{H}$. We denote such correspondence as $A \leftrightarrow |A\rangle\rangle$. When $A \leftrightarrow |A\rangle\rangle$ and $B \leftrightarrow |B\rangle\rangle$, one can obtain the correspondence (Arimitsu and Umezawa, 1987a)

$$AB \leftrightarrow |AB\rangle\rangle = A|B\rangle\rangle = \tilde{B}^\dagger |A\rangle\rangle. \tag{5}$$

The scalar product of vectors belonging to the Hilbert space $\mathscr{H} \otimes \tilde{\mathscr{H}}$ is given by the Hilbert-Schmidt product, that is, $\langle\langle A|B\rangle\rangle = \mathrm{Tr}(A^\dagger B)$. In NETFD, the average value of an observable A, including positive operator-valued measure which describes quantum measurement, in a quantum state ρ can be expressed as (Arimitsu and Umezawa, 1985, 1987a; Umezawa, 1993)

$$\langle A\rangle = \mathrm{Tr}[A\rho] = \mathrm{Tr}[\rho^{1-\alpha} A \rho^\alpha] = \langle\langle \rho^{1-\alpha}|A|\rho^\alpha\rangle\rangle, \tag{6}$$

with $0 \le \alpha \le 1$. A vector $|\rho^\alpha\rangle\rangle$ corresponding to a quantum state is invariant under the tilde conjugation, where the tilde conjugation of any vector is defined by $|\tilde{\Psi}\rangle\rangle = \sum_{m,n} c_{nm}^* |m, \tilde{n}\rangle\rangle$ for $|\Psi\rangle\rangle = \sum_{m,n} c_{mn} |m, \tilde{n}\rangle\rangle$. The tilde invariance of a state vector is equivalent to the Hermiticy of a density matrix. In NETFD with $\alpha = 1$, which is called the $\alpha = 1$ representation of NETFD, the average value of an observable A is given by $\langle A \rangle = \langle\langle 1|A|\rho\rangle\rangle$ while in the $\alpha = 0$ representation of NETFD, it is given by $\langle A \rangle = \langle\langle \rho|A|1\rangle\rangle$, where $|1\rangle\rangle$ is a vector corresponding to an identity operator. NETFD with $\alpha = 1/2$ is called the unitary representation (Umezawa, 1993).

When an open system obeys the quantum Markovian dynamics, the time-evolution of the quantum state $\rho(t)$ is determined by the quantum master equation of the Lindblad form,

$$\frac{\partial}{\partial t}\rho(t) = -\frac{i}{\hbar}[H, \rho(t)] + \frac{1}{\hbar}\sum_{\mu}\left[2A_\mu\rho(t)A_\mu^\dagger - A_\mu^\dagger A_\mu\rho(t) - \rho(t)A_\mu^\dagger A_\mu\right], \quad (7)$$

where H is the Hamiltonian of the system and A_μ is some operator of the system. In the $\alpha = 1$ representation of NETFD, the state vector $|\rho(t)\rangle\rangle$ obeys the Schrödinger-like equation,

$$\frac{\partial}{\partial t}|\rho(t)\rangle\rangle = -\frac{i}{\hbar}\hat{\mathscr{H}}|\rho(t)\rangle\rangle, \quad (8)$$

with

$$\hat{\mathscr{H}} = H - \tilde{H} + i\sum_{\mu}(2A_\mu\tilde{A}_\mu - A_\mu^\dagger A_\mu - \tilde{A}_\mu^\dagger\tilde{A}_\mu), \quad (9)$$

where these equations are derived from Eq. (7) by means of the relation (5). The time-evolution generator $\hat{\mathscr{H}}$ satisfies $\langle\langle 1|\hat{\mathscr{H}} = 0$ which is equivalent to the conservation law of probability. On the other hand, in the $\alpha = 0$ representation of NETFD, the time-evolution of the state vector $\langle\langle\rho(t)|$ is determined by

$$\frac{\partial}{\partial t}\langle\langle\rho(t)| = \frac{i}{\hbar}\langle\langle\rho(t)|\hat{\mathscr{H}}^\dagger, \quad (10)$$

with

$$\hat{\mathscr{H}}^\dagger = H - \tilde{H} - i\sum_{\mu}(2A_\mu^\dagger\tilde{A}_\mu^\dagger - A_\mu^\dagger A_\mu - \tilde{A}_\mu^\dagger\tilde{A}_\mu), \quad (11)$$

where the equality $\hat{\mathscr{H}}^\dagger|1\rangle\rangle = 0$ ensures the probability conservation. It is noted that although $\langle\langle 1|\hat{\mathscr{H}} = 0$ and $\hat{\mathscr{H}}^\dagger|1\rangle\rangle = 0$ always holds (Arimitsu and Umezawa, 1987a), $\hat{\mathscr{H}}|1\rangle\rangle = 0$ and $\langle\langle 1|\hat{\mathscr{H}}^\dagger = 0$ do not unless $[A_\mu, A_\mu^\dagger] = 0$. Furthermore, it is obvious that $\hat{\mathscr{H}}^\dagger \neq \hat{\mathscr{H}}$. In NETFD, the equation of motion for a state vector of an open system has the same form as that of a closed system. The difference is in the form of the time-evolution generator. For a closed system with a Hamiltonian H, the time evolution generator is given by $\hat{\mathscr{H}} = H - \tilde{H}$ and there is no cross terms of operators with and without the tilde and the equality $\hat{\mathscr{H}} = \hat{\mathscr{H}}^\dagger$ holds. On the other hands, for an open system, the time evolution generator $\hat{\mathscr{H}}$ includes the cross terms which is essential for describing an irreversible time-evolution and in general, $\hat{\mathscr{H}} \neq \hat{\mathscr{H}}^{\dagger}$. Since a state vector is always invariant under the tilde conjugation, the time-evolution generator $\hat{\mathscr{H}}$ must satisfy $(i\hat{\mathscr{H}})^{\sim} = i\hat{\mathscr{H}}$ (Arimitsu and Umezawa, 1987a). Then the time-evolution generator $\hat{\mathscr{H}}$ can be written in the form of $\hat{\mathscr{H}} = \hat{\mathscr{H}}_1 + i\hat{\mathscr{H}}_2$ with $(\hat{\mathscr{H}}_1)^{\sim} = -\hat{\mathscr{H}}_1$ and $(\hat{\mathscr{H}}_2)^{\sim} = \hat{\mathscr{H}}_2$. When an open system obeys the quantum Markovian dynamics, one finds that $\hat{\mathscr{H}}_1 = H - \tilde{H}$ and $\hat{\mathscr{H}}_2 = \sum_{\mu}(2A_{\mu}\tilde{A}_{\mu} - A_{\mu}^{\dagger}A_{\mu} - \tilde{A}_{\mu}^{\dagger}\tilde{A}_{\mu})$.

3. QUANTUM PREDICTION AND RETRODICTION

The probabilistic structure of quantum mechanics can be formulated in terms of two sets of positive operators. One is the set of preparation device (PD) operators $\{\Lambda_k\}$ and the other is the set of measurement device (MD) operators $\{\Gamma_j\}$ (Pegg *et al.*, 2002a,b). The joint probability $P(j, k)$ that a system is prepared in the k-th quantum state and quantum measurement performed on the system yields the j-th outcome is given by

$$P(j, k) = \frac{\mathrm{Tr}(\Gamma_j \Lambda_k)}{\mathrm{Tr}(\Gamma \Lambda)}, \tag{12}$$

where $\Lambda = \sum_k \Lambda_k$ and $\Gamma = \sum_j \Gamma_j$. Here the time-evolution of the system between the preparation and measurement has been ignored for the sake of simplicity. The predictive quantum state is defined by (Pegg *et al.*, 2002a,b)

$$\rho_k^{\mathrm{pred}} = \frac{\Lambda_k}{\mathrm{Tr}\Lambda_k}, \tag{13}$$

and the conditional probability $P(j|k)$ that the j-th measurement outcome is obtained when the system is prepared in the quantum state $\hat{\rho}_k^{\mathrm{pred}}$ is given by

$$P(j|k) = \frac{\mathrm{Tr}\left(\Gamma_j \rho_k^{\mathrm{pred}}\right)}{\mathrm{Tr}\left(\Gamma \rho_k^{\mathrm{pred}}\right)}. \tag{14}$$

From the causality, the MD operators must satisfy the relation $\Gamma = G \cdot 1$, where G is a positive constant (Pegg, 2006). In this case, the operator $\Pi_k = (1/G)\Gamma_k$ becomes the positive operator-valued measure describing the quantum measurement. Then one obtains the conventional expression of the conditional probability, that is, $P(j|k) = \mathrm{Tr}(\Pi_j \rho_k^{\mathrm{pred}})$, and thus one finds that ρ_k^{pred} is the usual density matrix which represents the quantum state of the system. On the other hand, the retrodictive quantum state of the system is defined by (Pegg *et al.*, 2002a,b)

$$\rho_j^{\mathrm{retr}} = \frac{\Gamma_j}{\mathrm{Tr}\Gamma_j}, \tag{15}$$

in term of which the posterior probability $Q(k|j)$ that the system was prepared in the k-th quantum state when the quantum measurement yields the j-th outcome is given by

$$Q(k|j) = \frac{\mathrm{Tr}\big(\Lambda_k \rho_j^{\mathrm{retr}}\big)}{\mathrm{Tr}\big(\Lambda \rho_j^{\mathrm{retr}}\big)}. \tag{16}$$

The probabilities $P(j|k)$ and $Q(k|j)$ are related with each other by the Bayes relation $P(j,k) = P(j|k)P(k) = Q(k|j)Q(j)$ (Barnett *et al.*, 2000b), where $P(k) = \sum_j P(j,k)$ and $Q(j) = \sum_k P(j,k)$.

In the $\alpha = 1$ representation of NETFD, the probabilistic structure of quantum mechanics is reformulated as follows. Let $|\Lambda_k\rangle\rangle$ and $|\Gamma_j\rangle\rangle$ be state vectors which correspond to the PD operator Λ_k and the MD operator Γ_j. Then the joint probability $P(j,k)$ that the system is prepared in the k-th quantum state and quantum measurement yields the j-th outcome is given by

$$P(j,k) = \frac{\langle\langle 1|\Gamma_j|\Lambda_k\rangle\rangle}{\langle\langle 1|\Gamma|\Lambda\rangle\rangle} = \frac{\langle\langle 1|\Lambda_k|\Gamma_j\rangle\rangle}{\langle\langle 1|\Lambda|\Gamma\rangle\rangle}, \tag{17}$$

where $|\Gamma\rangle\rangle = \sum_j |\Gamma_j\rangle\rangle$ and $|\Lambda\rangle\rangle = \sum_k |\Lambda_k\rangle\rangle$. Furthermore the predictive and retrodictive quantum states in NETFD can be represented by

$$|\rho_k^{\mathrm{pred}}\rangle\rangle = \frac{|\Lambda_k\rangle\rangle}{\langle\langle 1|\Lambda_k\rangle\rangle}, \tag{18}$$

$$|\rho_j^{\mathrm{retr}}\rangle\rangle = \frac{|\Gamma_j\rangle\rangle}{\langle\langle 1|\Gamma_j\rangle\rangle}, \tag{19}$$

in terms of which the conditional probability $P(j|k)$ and the posterior probability $Q(k|j)$ are given by

$$P(j|k) = \frac{\langle\langle 1|\Gamma_j|\rho_k^{\mathrm{pred}}\rangle\rangle}{\langle\langle 1|\Gamma|\rho_k^{\mathrm{pred}}\rangle\rangle}, \tag{20}$$

$$Q(k|j) = \frac{\langle\langle 1|\Lambda_k|\rho_j^{\mathrm{retor}}\rangle\rangle}{\langle\langle 1|\Lambda|\rho_j^{\mathrm{retor}}\rangle\rangle}. \tag{21}$$

Note that Eq. (20) becomes $P(j|k) = \langle\langle 1|\Pi_j|\rho_k^{\mathrm{pred}}\rangle\rangle$ due to the causality.

4. RETRODICTIVE TIME-EVOLUTION IN NETFD

Since the time-evolution of an open system is given by the same form as that of a closed system in NETFD, the retrodictive time-evolution of an open system can easily be obtained. Let $\hat{\mathscr{H}}$ be the time-evolution generator for a predictive quantum state in the $\alpha = 1$ representation of NETFD. When an open system is initially prepared in a quantum state $|\rho_k^{\text{pred}}\rangle\rangle$ at time t_p, the quantum state just before the measurement performed at time t_m is given by

$$|\rho_k^{\text{pred}}(t_m)\rangle\rangle = e^{-(i/\hbar)\hat{\mathscr{H}}(t_m - t_p)}|\rho_k^{\text{pred}}\rangle\rangle = \frac{|\Lambda_k(t_m)\rangle\rangle}{\langle\langle 1|\Lambda_k(t_m)\rangle\rangle}, \tag{22}$$

with

$$|\Lambda_k(t_m)\rangle\rangle = e^{-(i/\hbar)\hat{\mathscr{H}}(t_m - t_p)}|\Lambda_k\rangle\rangle. \tag{23}$$

In Eq. (22), the equality $\langle\langle 1|\hat{\mathscr{H}} = 0$ has been used. Then the conditional probability $P(j|k)$ that the measurement performed on the system at later time t_m yields the j-th outcome is given by

$$P(j|k) = \frac{\langle\langle 1|\Gamma_j|\rho_k^{\text{pred}}(t_m)\rangle\rangle}{\langle\langle 1|\Gamma|\rho_k^{\text{pred}}(t_m)\rangle\rangle}, \tag{24}$$

or equivalently $P(j|k) = \langle\langle 1|\Pi_j|\rho_k^{\text{pred}}(t_m)\rangle\rangle$. Furthermore the joint probability $P(j,k)$ that the system is prepared in the k-th quantum state at the time t_p and the k-th measurement outcome is obtained at the time t_m is expressed as

$$P(j,k) = \frac{\langle\langle 1|\Gamma_j|\Lambda_k(t_m)\rangle\rangle}{\langle\langle 1|\Gamma|\Lambda(t_m)\rangle\rangle} = \frac{\langle\langle 1|\Gamma_j e^{-(i/\hbar)\hat{\mathscr{H}}(t_m - t_p)}|\Lambda_k\rangle\rangle}{\langle\langle 1|\Gamma e^{-(i/\hbar)\hat{\mathscr{H}}(t_m - t_p)}|\Lambda\rangle\rangle} \tag{25}$$

which can be rewritten in the following form:

$$P(j,k) = \frac{\langle\langle 1|\Lambda_k|\Gamma_j(t_p)\rangle\rangle}{\langle\langle 1|\Lambda|\Gamma(t_p)\rangle\rangle}, \tag{26}$$

with

$$|\Gamma_j(t_p)\rangle\rangle = e^{(i/\hbar)\hat{\mathscr{H}}^{\dagger}(t_m - t_p)}|\Gamma_j\rangle\rangle. \tag{27}$$

This implies that the retrodictive state $|\rho_j^{\text{retr}}(t_p)\rangle\rangle$ at the time t_p just after the preparation is given by

$$|\rho_j^{\text{retr}}(t_p)\rangle\rangle = \frac{|\Gamma_j(t_p)\rangle\rangle}{\langle\langle 1|\Gamma_j(t_p)\rangle\rangle}. \tag{28}$$

Here $\langle\langle 1|\Gamma_j(t_p)\rangle\rangle \neq \langle\langle 1|\Gamma_j\rangle\rangle$ in general. The posterior probability $Q(k|j)$ that the system was in the k-th quantum state once the j-th measurement outcome is obtained can be expressed as

$$Q(k|j) = \frac{\langle\langle 1|\Lambda_k|\rho_j^{\text{retr}}(t_p)\rangle\rangle}{\langle\langle 1|\Lambda|\rho_j^{\text{retr}}(t_p)\rangle\rangle}. \tag{29}$$

We now consider the equations of motion for the PD vector $|\Lambda_k\rangle\rangle$, the predictive state vector $|\rho_k^{\text{pred}}\rangle\rangle$, the MD vector $|\Gamma_j\rangle\rangle$ and the retrodictive state vector

$|\rho_j^{\mathrm{retr}}\rangle\rangle$. It is obvious from Eqs. (22) and (23) that the PD vector $|\Lambda_k(t)\rangle\rangle$ and the predictive state vector $|\rho_k^{\mathrm{pred}}(t)\rangle\rangle$ obeys

$$\frac{\partial}{\partial t}|\Lambda_k(t)\rangle\rangle = -\frac{i}{\hbar}\hat{\mathscr{H}}|\Lambda_k(t)\rangle\rangle, \tag{30}$$

$$\frac{\partial}{\partial t}|\rho_k^{\mathrm{pred}}(t)\rangle\rangle = -\frac{i}{\hbar}\hat{\mathscr{H}}|\rho_k^{\mathrm{pred}}(t)\rangle\rangle, \tag{31}$$

where the initial conditions are given at the time t_p. Thus the time-evolution generators of the PD vector and predictive state vector are equal to that in the $\alpha = 1$ representation of NETFD. On the other hand, from Eq. (27), the MD vector $|\Gamma_k(t)\rangle\rangle$ is subject to

$$\frac{\partial}{\partial t}|\Gamma_j(t)\rangle\rangle = -\frac{i}{\hbar}\hat{\mathscr{H}}^{\dagger}|\Gamma_j(t)\rangle\rangle, \tag{32}$$

where the "initial" condition is given at the time t_m. This result implies that the time-evolution generator of the MD vector is identical with that in the $\alpha = 0$ representation of NETFD. The equation of motion for the retrodictive state vector $|\rho_j^{\mathrm{retr}}(t)\rangle\rangle$ is somewhat complicated due to the normalization factor,

$$\frac{\partial}{\partial t}|\rho_j^{\mathrm{retr}}(t)\rangle\rangle = -\frac{i}{\hbar}\Delta\hat{\mathscr{H}}_j^{\mathrm{retr}}(t)|\rho_j^{\mathrm{retr}}(t)\rangle\rangle, \tag{33}$$

with

$$\Delta\hat{\mathscr{H}}_j^{\mathrm{retr}}(t) = \hat{\mathscr{H}}^{\dagger} - \langle\langle 1|\hat{\mathscr{H}}^{\dagger}|\rho_j^{\mathrm{retr}}(t)\rangle\rangle. \tag{34}$$

The time flows backward from the future to the past in Eqs. (32) and (33). Note that one can formally rewrite Eq. (31) in the same form as Eq. (33),

$$\frac{\partial}{\partial t}|\rho_k^{\mathrm{pred}}(t)\rangle\rangle = -\frac{i}{\hbar}\Delta\hat{\mathscr{H}}_k^{\mathrm{pred}}(t)|\rho_k^{\mathrm{pred}}(t)\rangle\rangle, \tag{35}$$

with

$$\Delta\hat{\mathscr{H}}_k^{\mathrm{pred}}(t) = \hat{\mathscr{H}} - \langle\langle 1|\hat{\mathscr{H}}|\rho_j^{\mathrm{pred}}(t)\rangle\rangle = \hat{\mathscr{H}}. \tag{36}$$

The time-evolution of the predictive state is determined by the time-evolution generator of the $\alpha = 1$ representation of NETFD. On the other hand, although when the $\alpha = 1$ representation of NETFD is applied, the time-evolution of the retrodictive state is determined by the time-evolution generator of the $\alpha = 0$ representation. This result may be related with the causality in NETFD. The $\alpha = 1$ representation of NETFD provides the average values of the time-ordered products of operators while in the $\alpha = 0$ representation, the average value of the anti-time-ordered products are obtained (Umezawa, 1993). When the dynamics of the system is subject to the quantum Markovian process, the predictive time-evolution generator $\hat{\mathscr{H}}$ is given by Eq. (9). Then the corresponding retrodictive time-evolution generator $\hat{\mathscr{H}}^{\dagger}$ is given by Eq. (11) and $\Delta\hat{\mathscr{H}}_j^{\mathrm{retr}}(t)$ is calculated to be

$$\Delta\hat{\mathscr{H}}_j^{\mathrm{retr}}(t) = \hat{\mathscr{H}}^{\dagger} + 2i \sum_{\mu} \langle\!\langle 1 | [A_\mu, A_\mu^{\dagger}] | \rho_j^{\mathrm{retr}}(t) \rangle\!\rangle, \tag{37}$$

which is equivalent to that obtained directly from the Markovian master equation of the Lindblad form in the conventional formalism (Pegg *et al.*, 2002b).

Since the equality $\hat{\mathscr{H}}^{\dagger}|1\rangle\rangle = 0$ always holds, the equation of motion for the MD vector $|\Gamma_j\rangle\rangle$ has a stationary solution $|\Gamma_j\rangle\rangle = |1\rangle\rangle$. If the system is described by N dimensional Hilbert state, the retrodictive quantum state in the time region with $t_m - t_p \to \infty$ is given by $|\rho_j^{\mathrm{retr}}(t_p)\rangle\rangle = (1/N)|1\rangle\rangle$ which represents the completely random state. It is easy to check that $\Delta\hat{\mathscr{H}}_j^{\mathrm{retr}}(t)|\rho_j^{\mathrm{retr}}(t)\rangle\rangle|_{|\rho_j^{\mathrm{retr}}(t)\rangle\rangle\to(1/N)|1\rangle\rangle} = 0$. This result means the fact that the information on the measurement outcome obtained at the time t_m is lost completely during the retrodictive time evolution. It is important to note that $\hat{\mathscr{H}}^{\dagger}|1\rangle\rangle = 0$ is equivalent to $\langle\langle 1|\hat{\mathscr{H}} = 0$ which ensures the probability conservation during the predictive time-evolution. Hence missing the information on the measurement outcome during the retrodictive time-evolution results from the probability conservation during the predictive time-evolution. The probability is conserved during the retrodictive time-evolution since equality $\langle\langle 1|\Delta\hat{\mathscr{H}}_j^{\mathrm{retr}}(t)|\rho_j^{\mathrm{retr}}(t)\rangle\rangle = 0$ alway holds.

5. CONCLUDING REMARKS

In this paper, the retrodictive time-evolution of an open system which obeys the quantum Markovian process has been formulated by means of NETFD. Since NETFD can treat closed and open systems in the same way, the retrodictive time-evolution generator can easily be obtained. It is important to note that the derivation of the retrodictive time-evolution generator is restricted to the quantum Markovian dynamics. The derivation of the retrodictive time-evolution generator for the quantum non-Markovian dynamics is difficult. Such time-evolution generator can, however, be derived by means of the time-convolutionless projection operator method (Shibata and Arimitsu, 1980; Arimitsu, 1982; Uchiyama and Shibata, 1999). This will be published elsewhere.

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